HIGH-DIMENSIONAL NETWORKS AND THEIR PERSISTENT HOMOLOGY

Instructor: Simona Settepanella[1]

Duration: 18h

Course Description. The goal of this course is to provide students with:

(1) a minimum vocabulary of the nowadays Discrete Mathematics (Combinatorics);
(2) a basic introduction to graphs, networks (a special type of graphs) and cell complexes which are the higher dimensional generalization of graphs;
(3) the knowledge of one of the main algebraic tool to study cell complexes: their (persistent) homology;
(4) a basic knowledge of computational tools (e.g. softwares) available to compute cell complexes persistent homology.

By the end of the course students will be able to orient inside the most recent discrete mathematical tools available. They will be able to understand if and how cell complexes (and persistent homology) can be used as a tool to model and study problems in their own research field. Finally they will know existing softwares to compute the persistent homology of cell complexes.

The mathematics used to model problems in main stream economics is almost exclusively continuous mathematics. But since data are discrete, nowadays it becomes essential to have a minimum level of knowledge in discrete mathematics (that is combinatorics) which objects are matrices, graphs, cell complexes and, more in general, matroids (which include all the previous tools and more). Hence this course is open to all the Ph.D. students who want to have in their tool box a basic knowledge that could prove helpful in model the economics research problems they will approach in their careers.

No prior knowledge is required except basic algebra operations.

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Grading and Assessment.

- **In-class participation** (30%)**:** Active engagement in discussions and critical assessment of assigned readings.
- **Final exam** (70%)**:** A take-home assignment requiring students to propose a research question which can be modeled via cell complexes (or networks) and studied by mean of persistent homology. Group work is also accepted.

The final assignment due date will be negotiated with students.

Course Outline. The course will be divided in two parts.

Part I. Basic tools in Discrete Mathematics (nowadays called Combinatorics):

(1) Introductory concepts on matrices, graphs, networks (as special type of graphs), matroids and how they are related;
(2) A slightly deeper look into Graph Theory:
   (a) Distance in graphs: definitions and few properties;
   (b) Trees and spanning trees;
   (c) Trails, Circuits, Paths and Cycles: the Bridges of Konigsberg; Eulerian trails and Hamiltonian paths;
(3) Introductory concepts on Cell complexes: definition, main properties and examples;
(4) A special type of cell complexes: simplicial complexes.
(5) Examples of the use of networks and cell complexes to model problems in Economics.

Part II. (Persistent) Homology of Cell Complexes:

(1) Boundary maps in cell (simplicial) complexes; kernel and image of the boundary map;
(2) Homology of cell complexes: definition and properties;
(3) Betti's numbers and their properties;
(4) Example of computation of homology in case of graphs, networks and simplicial complexes;
(5) Persistent homology of cell complexes: definition and main properties;
(6) Example of applications of persistent homology of cell complexes in Neuroscience, Social Sciences and Economics (a selected number of published papers on the argument will be provided);
(7) Existing softwares and packages to compute persistent homology.

Bibliography. The bibliography and readings will be provided later on and during the course.